**Lesson: Use Kalman Filtering for Travel Time Estimation, Origin-Destination Demand Estimation, and Traffic Sensor Network Design**

**Authors:**

To design an accurate traveler information system, we need to reliably estimate travel time for limited traffic measurements. To design a transportation sensor network, the decision-maker needs to determine what sensor investments should be made, as well as when, how, where and with what technologies. This learning document focuses on three important traffic related Kalman filtering applications:

1. **travel time estimation on multiple links,**
2. **origin-destination demand estimation on a traffic network**
3. **traffic sensor location (and traffic information theory)**

**Learning Goals**

1. Understand application of Kalman Filter in OD flow estimation and travel time estimation
2. Understand major modeling components of KF: gain factor, measurement, P- and P+
3. Know how Kalman filter is derived as an optimization model
4. Interpret uncertainty matrix, trace and determine
5. Know how to apply the concept of Kalman filter in sensor network design
6. **Overview of Kalman filtering and sensor location problem**

We will discuss how a Kalman filter is implemented for travel time estimation applications, and how to locate limited set of traffic counting stations and automatic vehicle identification readers in a network so as to maximize the expected information gain for the subsequent origin-destination demand estimation problem.

* 1. **Notations**

We first introduce all the sets and subscripts in the sensor location and OD flow estimation problems.

**Sets**

*I* = set of origin zones,

*J* = set of destination zones,

*L* = set of links,

 = set of links with point observations (e.g. link counts), ,

= set of links with point-to-point observations (e.g. vehicle identification counts), ,

 = set of links with sensors, .

**Size of sets**

*m =* number of observations,

*n* = number of OD pairs |*I*|×|*J*|,

*q* = number of nodes in a sensor network.

**Subscripts**

 = subscript for origin/destination zone, *i*∈*I*, *j*∈*J*,

 = subscript for link with traffic measurements,

 = subscript for sensor sequence,

 *=* subscript for measurement used in Kalman filter updating,

**Estimation variables**

*d*(*i*,*j*) = OD flow volume with destination in zone *j*, originating their trips from zone *i*.

**Measurements**

 = number of vehicles passing through link *l*,

*=* number of tagged vehicles passing through link *l*,

**Mapping matrices**

 = link flow proportions, i.e. proportion of vehicular OD flows from origin *i* to destination *j*, contributing to the flow on link *l*,

**Estimation error terms**

*ωl* = measurement error on link *l* related to senor equipment and environment,

**Vector and matrix forms in Kalman filtering framework**

*C* = sensor measurement vector, consisting of *m* elements,

*D* = OD flow vector, consisting of *n* elements *d*(*i*,*j*),

 = initial OD flow vector used for generating link proportions through traffic assignment,

= *a priori* estimate of the mean values in the demand vector, consisting of *n* elements,

= *a posteriori* estimate of the mean values in the flow vector,

 = *a posteriori* flow estimate error, i.e. ,

= *a priori* error covariance matrix of flow estimate, consisting of (*n*×*n*) elements,

= *a posteriori* error covariance matrix, i.e. conditional covariance matrix of estimation errors after including measurements,

*H* = sensor matrix that maps unknown flows *D* to measurements *C*, consisting of (*m*×*n*) elements,

*K* = updating gain matrix, consisting of (*n*× *m*) elements,

*R* = variance covariance matrix for combined errors, including measurement and modeling errors,

*ε* = combined error term, .

Consider a traffic network with multiple origins *i*∈*I* and destinations *j*∈*J*, as well as a set of nodes connected by a set of directed links. Given prior information on OD trips, the sensor location problem seeks to find a set of links = so that link counts *c*'*l* are available on link  and vehicle identification data are available from AVI readers located on link  for the subsequent OD flow estimation problem.

The goal of the sensor location problem is to maximize information gain from the sensor set on , subject to budget constraints for installation and maintenance.

**1.2 Risk function or objective function for estimation**

One of the fundamental questions in sensor location problems is which criteria should be selected to drive the underlying optimization processes. Essentially, a typical estimation problem is to find a new estimate  that can combine and utilize information from prior estimates and sensor measurements. By definition, the posterior error covariance matrix is

. (1)

If the estimator is unbiased, then the above equation reduces to

. (2)

**1.3 Two different estimators**

In the following, we examine two commonly used estimation criteria, namely, the mean-square error and entropy. The classic Kalman filter aims to minimize the mean-square error, that is, the Euclidean norm square of:

, (3)

and equals the trace of the variance and covariance matrix:

. (4)

Entropy is another commonly used measure of information. For a discrete variable, Shannon’s original entropy is defined as the number of ways in which the solution could have arisen. For a continuously distributed random vector *D*, on the other hand, the entropy is measured by , where *f* is the joint density function for *D*. If *D* follows a normal distribution, then its entropy is quantified as

, (5)

where  is a constant that depends on the size of *D*, for example, the number of unknown OD pairs in the context of OD flow estimation. The entropy measure is proportional to the log of the determinant of the covariance matrix. By ignoring the constant  and the monotonic logarithm function, we can simplify the entropy-based information measure for the posterior estimate as .

**1.4. Geometric interpretation**

Geometrically, the determinant of a variance covariance matrix can be interpreted as a measure of the volume of a hyperellipsoid for unknown estimation variables centered at , that is, the axis directions of the ellipsoid are given by the eigenvectors of , and the lengths of the axes are proportional to the square root of the eigenvalues. The solid ellipsoid of D values satisfying

 (6)

has a probability of 1 -, where  is the upper (100×)th percentile of a Chi-square distribution with *n* (i.e. number of OD pairs) degree of freedom. The detailed description of (6) can be found in Bryson and Ho (1975).

C:\Documents and Settings\zhou\My Documents\My_own_documents\My papers\Sensor Network Design_for_OD_estimation\final_package\Figure 2 Likelihood ellipses for a demand vector of two OD pairs.tif

**Figure 1 Likelihood ellipses for a demand vector of two OD pairs**

In comparison, the trace of a covariance matrix, which is used in the mean-square criterion, corresponds to the circumference of the rectangular region that encloses the ellipsoid. Both trace and determinant measures, in fact, use single numerical values to describe the amount of variations in random variables. Moreover,  and  are, respectively, the sum and product of the eigenvalues associated with the covariance matrix .

As an illustration, Fig. 1 shows likelihood ellipses for a demand flow vector of two OD pairs with the same mean vector [3, 2] for the OD pair *d*1,2 and *d*1,3, and covariance matrices,  and , respectively. These three matrices correspond to the same trace value of 5 but different determinants of 4 and 3. Clearly, the trace function only utilizes the variance information, while the determinant function captures the correlation among the random variables. It should be remarked that, as single-valued measures, both the trace and determinant functions are unable to detect and distinguish different correlation structures. For example, Figs. 1-(a) and 1-(c) have the same trace and determinant values.

* 1. **Measurement matrix for OD demand estimation problem**

A linear measurement equation is used to relate the unknown estimation variable to measurements:

, where. (7)

In OD estimation problems, measurement vector *C* includes both link counts, and the size of *C* depends on the set of sensor locations. Sensor matrix *H* provides a linear mapping between OD flow flows and observations, and a typical example is a link flow proportion matrix that maps OD flows to link counts.

In short, the given conditions for the demand estimation problem can be mathematically summarized as: (1) the mean  and covariance matrix of the *a priori* flow vector, (2) the measurement error covariance matrix *R* and the sensor matrix *H* for all possible sensor sites.

The measurement equation for using link counts is typically expressed as:

. (8)

That is, the link flow count on link *l* is the sum of flow passing through link *l* from different OD pairs plus a measurement error. Since it is difficult to directly measure the true values of link flow proportions, especially in a congested traffic network, the analyst typically uses traffic assignment or simulation programs to produce an estimate of the link proportions.

|  |
| --- |
| The optimal form of *K* is then derived as    Under the optimal formulation of the Kalman gain matrix in the following equation, a simplified expression for the estimation error covariance is derived as    Other formulas of the estimation error covariance are available, for example, |

**1.6 Kalman Filtering for Travel Time Estimation**



Table 1. Calculation results of single-link example

|  |  |  |  |
| --- | --- | --- | --- |
|  | Prior estimate | Measurement | Posterior estimate |
| Travel time (min) |  |  |  |
| Variance (min2) |  |  |  |

Consider a single link shown in the above figure, where a link from *a* to *b* corresponding to sensor mapping matrix *H*=1. In the historical travel time database, the estimated travel time follows a normal distribution with a mean of 15 minutes and a standard deviation of 5 minutes (i.e . ). Given a new measurement  minutes with a measurement error variance of 5 minutes, we can calculate the optimal Kalman filtering gain factor as



Then the travel time estimate is updated by

 ,

and the posterior estimation variance is reduced to

.

The calculation results are summarized in Table 1.

**Example Problem 1: Travel Time Estimation on a Single Link**

Utah DOT installed a new loop detector sensor on I-80 to measure the traffic of a 10-mile highway segment. The historical travel time of the segment is 10 min at 1 pm, with a measurement error variance of 16 min2. This model of sensor has a measurement variance of 4 min2 when functioning and well calibrated. However, this type of inductive loop detector has a high failure rate, and statistics shows that the measurement from a broken sensor has an error variance of 25 min2. The UDOT plans to collect data from this sensor every 5 min.

Please answer the following questions:

1. How to model the travel time estimation problem with the Kalman Filtering framework?
2. After a first measurement of 15 min with sensor working properly at 1 pm, what are the posterior estimates of travel time and variance?
3. If the surveillance camera footage at that area indicated that the sensor could be broken, what are the posterior estimates of travel time and variance after a first measurement of 18 min?
4. If UDOT models travel time update between 1 pm to 3 pm with equations *Xt+1 = Xt* + *wt*, *wt ~ N* (0*, Q*) and Q = [1], how to predict the priori estimates of travel time and variance at 1:05 pm?

**Solutions:**

1. This single-link network can be modeled with matrix H = [1]. The travel time estimation model is:



and under optimal Kalman gain function  , the variance estimation model is:



The prediction model is:





1. With sensor working properly:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Time interval | *t* | | | *t*+1 |
|  | Priori estimate | Measurement | Posteriori estimate | Priori estimate |
| Travel time (min) |  |  |  |  |
| Variance (min2) |  |  |  |  |











With a working sensor, the estimate is closer to the measurement than to the priori estimate.



1. With sensor broken:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Time interval | *t* | | | *t*+1 |
|  | Priori estimate | Measurement | Posteriori estimate | Priori estimate |
| Travel time (min) |  |  |  |  |
| Variance (min2) |  |  |  |  |











With large measurement error *R*, the posteriori estimate is closer to the priori estimate



**Example Problem 2: Travel Time Estimation on Two Links: Using Excel to implement Kalman filter**

**Example File: [lesson 9.1\_learning\_KalmanFiltering.xlsx]**

The associated Excel file aims to teach you how to use Kalman filtering to estimate link travel times from sensor data. In particular, this document shows how to update the link travel time by Kalman filtering if new travel time measurements are received, with respect to the *a priori* link travel time estimate and the estimation correlation matrix. New travel time measurements can come from point sensors, such as loop detectors, or point-to-point sensors, such as GPS traces.

***Main Variables and Equations***

The state variable is defined as the travel time on each link *i*, where *N* is the number of the links in the network. **In the example shown in the Excel file, the link travel time on link *a* and link *b* is the state variables: where x1 is the link travel time on link *a* and x2 is the link travel time on link *b*.**

The measurement *Z* is linear equations of link travel time with regard to the measurement errors.

 (E.1)

where R is the measurement error matrix and H is the mapping matrix relating the link travel time to the measurement. **In the Excel file, there are two types of measurements: point measurements and point-to-point (AVI) measurements. In the example, there are two point observations on both link *a* and link *b*. Therefore, this measurement *Z1* is represented as a linear combination of two link travel time and the associated measurement errors.**



**As we can see, the mapping matrix**

**In the case of point-to-point measurement, the measurement *Z2* is represented as another link combination of link travel time and the measurement error.**

****

**Here, the mapping matrix.**

The equations of Kalman filter fall into two groups: *time update* equations and *measurement update* equations. The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the *a priori* estimates for the next time step. The measurement update equations are responsible for the feedback—i.e. for incorporating a new measurement into the *a priori* estimate to obtain an improved *a posteriori* estimate.

In our case, the time update equations are:

 (E.2)

 (E.3)

where is the updated estimation covariance matrix, is the estimated estimation covariance matrix, and Q is the process error matrix. and are the *a priori* estimates. Remark, the system update equation (E.2) in this case is just a simple projection from the current state. It can be further extended to other models.

In each time step, we need first to calculate the Kalman gain K (skip the time step index *t*) so as to update the estimation from the measurement:

 (E.4)

Then, update the link travel time estimate with the measurement by

 (E.5)

and the estimation covariance matrix by

 (E.6)

and are the *a posteriori* estimates.

***Illustrated Kalman Filtering Process***

In the Excel file, *a priori* link travel time estimate  and the estimation covariance matrix are given.

When a new point measurement is received, the Kalman gain *K* is first calculated by eq. (E.4). In this case, ,and (it is assumed that there is no correlation between these two observations).



Then, the link travel time estimate and the estimation covariance matrix are updated by eq. (1.5) and eq. (E.6). ():



.

After the estimation update process, the link travel time is predicted forward. As mentioned before, the predicting model in this example is a simple one as shown in eq. (E.2). Furthermore, it is assumed there is no process error so that Q=0. Thus, and .

**Advanced Material: Derivation of Kalman filtering and Application in Traffic Sensor Network Design**

**3. Derivation of Kalman filter**

First, we assume the estimator takes a linear updating form of

. (9)

Essentially, the term is the error of the prior estimate, which is also known as the innovation residual or measurement residual. The above equation can be viewed as the update phase in Kalman filtering, in which measurement information from the current stage is used to correct/refine the *a priori* estimate  to obtain a new estimate.

Substituting Eq. (9) into gives

. (10)

Substituting *C* from the linear sensor equation (7) into the above equation, we have

. (11)

Assuming the measurement error term  is uncorrelated with the other terms leads to

. (12)

where *R* is the covariance matrix of the measurement errors. The above formula describes the propagation of error covariances for any given updating matrix *K*. To minimize the trace of the posterior variance-covariance matrix, we need to find an optimal matrix *K* that satisfies

. (13)

Solving the above matrix derivative equation for *K*, the optimal weighting matrix is

. (14)

Substituting the above optimal gain matrix back into Eq. (22) leads to

. (15)

It is worth noting that many equivalent formulations exist for the above equation, for example,

. (16)

To show the equivalency of Eq. (15) and Eq.(16), one can apply the Matrix Inversion Lemma (MIL) as follows:

.

Recall that, the matrix inversion lemma is , where , , and  in our case.

Eq. (16) is commonly used to illustrate how the information is accumulated and updated in the Kalman filter.

This simple additive form updates the prior belief state  by a linear combination of observation information. In the case of a complete lack of historical demand information, we can set .

The error covariance updating equations (15) and (16) clearly show the linkage between the *a priori* uncertainty and the *a posteriori* uncertainty. *KH* in Eq. (15) measures the degree of uncertainty reduction due to inclusion of new measurements, while  in Eq. (16) corresponds to the value of additional information from sensors.

For the sensor location problem, the most important and useful property of the above linear mean square error updating formula is that the posterior covariance matrix  is independent of the specific value of the measurements *C*, although the conditional mean estimate  is determined by the detailed values of sensor data *C*. As a result, we can calculate  and the expected information gain *before* installing any sensors and obtaining measurements from them.

Let *Hk* be the *k*th row vector of matrix *H*, corresponding to the *k*th measurement. If measurement errors are uncorrelated, then *R* = diag{*rk*} and it is easy to show that

. (17)

Similar to Eq. (16), the above error covariance updating equation is commonly used in the information form of the Kalman filter, in which the information matrix  is recursively updated by the incoming sensor information from measurement *k*. Recall that the product of  is an *n*×*n* matrix, where *n* is the number of OD pairs. For OD pairs that have flows passing through the selected sensor links, the corresponding link proportion is positive. As a result, different sensor locations could lead to positive values at the different cells in the variance covariance matrix of posterior OD flow estimates, where these cells correspond to the OD pairs traversing the sensor links. In a later section, Fig. 3 illustrates how different single sensors affect the OD estimation uncertainty; Fig. 6 shows how multiple sensors could improve the overall estimation quality. Under the measurement error independence assumption, one can avoid matrix inversion in calculating the gain matrix *Kk* for the *k*th measurement by using the sequential updating formula

. (18)

**4. Understand Kalman filter from sensor location perspective**

Because the sensor location stage needs to incorporate as much sensor information as possible, the importance of a sensor at a certain location depends on the *value of the information/knowledge* that it can provide for the OD estimation stage. In light of Eq. (`7), several key factors affect the possible information gain as shown in the following.

**Sensor error:** A large combined error variance *R* yields a small increase in  and accordingly a small reduction in OD flow estimation uncertainty in terms of . On the other hand, sensors with less noise allow us to find an OD flow estimate with greater accuracy.

**Sensor coverage:** If an OD pair is measured by at least one sensor, then the diagonal element associated with that OD pair in matrix  should be positive. For instance, in the case of point sensors, the diagonal element for OD pair (*i*,*j*) is . If a set of sensors covers all the OD pairs in a network, then we have positive values for all the diagonal elements. If  has full rank, then its inverse exists and we can obtain a unique estimate even without prior information (i.e.). It should be noted that positive diagonal elements in matrixdo not imply that the matrix has full rank (e.g.  in a two OD pair case).

**Marginal information gain:** To obtain a meaningful marginal information gain with respect to the prior OD flow estimate, one should not only focus on finding a sensor matrix with adequate information, but also seek to ensure that the information content of the sensor matrix reduces the existing demand uncertainty. Consider an OD pair (*i*,*j*) with high uncertainty in the historical OD flow; that is, the diagonal element for OD pair (*i*,*j*) in is small. In this case, even a minor diagonal value for OD pair (*i*,*j*) in  could generate a considerable uncertainty reduction in the final variance matrix . In contrast, if the prior variance of OD pair (*i*,*j*) is already very small, a large amount of information from  does not necessarily produce a significant marginal quality improvement.

**Illustrations on Sensor Network Design**

We first use a series of examples in a 6-node network to demonstrate the proposed methodology. In Fig. 2, OD pair 1 travels from node 1 to node 2 and OD pair 2 travels from node 1 to node 3. The first OD pair has two routes. 70% of the flow travels along path {1, 4, 5, 2} while the remaining 30% travels along path {1, 4, 6, 5, 2}. Both OD pairs have a flow volume of 20 units. Let us assume , meaning that OD pair 1 has a larger *a priori* variance than OD pair 2. We first consider three alternative one-sensor location scenarios that do not involve estimation of the route choice percentages: scenario (a) covers OD pair 1, scenario (b) covers OD pair 2, and scenario (c) covers both OD pairs. The measurement matrices for the above three cases are, respectively, [1,0], [0,1] and [1,1], where the row in matrix *H* refers to a measurement and the column refers to an OD pair.

**C:\Documents and Settings\zhou\My Documents\My_own_documents\My papers\Sensor Network Design_for_OD_estimation\final_package\Figure 3 Examples of single point sensor location scenarios.tif**

**Figure 2** Examples of single point sensor location scenarios

For simplicity, we assume the standard deviation of the measurement error for a point sensor is 5% of the corresponding true flow volume. Fig. 2 shows the resulting sensor information matrices, posterior variance matrices, traces and determinates. It is interesting to note that the sensor information matrix for scenario (c) is , which contains considerable correlation between the estimate errors of the two OD pairs. The trace values for the three scenarios are 1.8, 4.5 and 3.11, which quantify different magnitudes of uncertainty reduction compared to the original  of 5. Specifically, scenario (a) allocates the scarce sensor resource to the critical OD pair with the largest variance, producing a better total uncertainty reduction than scenarios (b) and (c). In contrast, although scenario (c) fully covers both OD flows, it does not produce the maximum information gain in terms of both trace and determinant criteria. Actually, even if we assume *R*=1 in scenario (c), which is identical to scenarios (a) and (b), the resulting  and the trace and determinant are 2.16 and 0.66, respectively. These two values are still larger than the measures in scenario (a). The examples above reveal that the sensor location problem is more complicated than a simple network coverage problem, and that it is important to recognize and capture the uncertainties in the prior OD demand estimates and the available measurements.

**C:\Documents and Settings\zhou\My Documents\My_own_documents\My papers\Sensor Network Design_for_OD_estimation\final_package\Figure 4 Examples of single point sensor location scenarios with route choice.tif**

Figure 3 Examples of single point sensor location scenarios with route choice

Fig. 3 considers a more general case, in which the route choice percentage (i.e. link proportion) associated with link (4, 5) needs to be estimated from either traffic assignment programs or AVI data. Assuming there is no error in the flow proportion estimate, scenario (d) constructs a lower bound for the uncertainty reduction attainable by locating a point sensor on link (4, 5). It should be noted that the above benchmark scenario gives exactly the same posterior variance and covariance matrix as scenario (a) in Fig. 2. This indicates that if link flow proportions are error-free and the standard deviation of the measurement error is proportional to the link volume, then locating a sensor on a low-volume link can still extract significant OD demand information. In scenario (e), we assume that the standard deviation of measurement errors in the link proportion matrix (due to errors from traffic assignment) is 0.3. Clearly, a considerable amount of information loss is observed by comparing scenarios (d) and (e), as the corresponding trace value increases from 1.8 to 2.35. We assume that, by utilizing AVI counts to calibrate the link proportion, scenario (f) significantly reduces the standard deviation of the measurement error to 0.1. As a result, the overall OD demand estimation accuracy is considerably improved, while the associated trace value is only greater than the lower bound in scenario (d) by 10%.

**C:\Documents and Settings\zhou\My Documents\My_own_documents\My papers\Sensor Network Design_for_OD_estimation\final_package\Figure 5 Examples of multiple point sensor location scenarios.tif**

**Figure 4 Examples of multiple point sensor location scenarios**

Scenarios (h) to (l) in Fig. 5 examine a wide range of scenarios with two sensors. Scenarios (h) and (i) still focus on individual OD pairs, while scenarios (j) and (k) cover both OD pairs. Scenario (l) assumes that the measurement errors between the two sensors are correlated to each other. Overall, scenario (j) is able to systematically balance the information needs of the different OD pairs. Interestingly, while both scenarios (j) and (k) cover two OD pairs, scenario (j) dominates scenario (k) in the sense that the former provides a smaller value for each element in the variance and covariance matrix. Comparing scenarios (h) and (l), as expected, correlated measurement errors in the latter case lead to some information loss. Scenario (m) considers three inexpensive roadside sensors with relatively large levels of measurement noise. If the total cost of these three sensors is lower than the cost of the two expensive and accurate sensors in scenario (j), then scenario (m) is actually a more cost-efficient and reliable alternative. Again, these examples show the advantage of the proposed methodology in systematically evaluating the trade-off between the accuracy of individual sensors and network-wide reliability.

**5. Illustrative example**

In Fig. 5, we present an illustrative example with a 6-node hypothetical transportation network to demonstrate how the proposed measures of information can systematically evaluate the trade-offs between the accuracy and placement of individual AVI sensors for path travel time estimation reliability. This example focuses on the measure of estimation uncertainty with heuristic approach. As a comparison, the results of the proposed analytical method are also provided with lower bound evaluation. In addition, subscript time interval *t* is omitted for simplicity.

As shown in the base case, there are three traffic analysis zones at nodes *a*, *d* and *b*, and three major origin-to-destination trips: (1) *a* to *b*, (2) *a* to *d* and (3) *d* to *b*, each with a unit of flow volume.  (e.g., obtainable from a historical travel time database with point detectors) leads to a trace of 12 and a determinant of 48. Among the 5 links along the corridor, link 5 from node *f* to *b* has the highest uncertainty in terms of link travel time estimation variance. We can view node *b* as a downtown area, and the incoming flow from the other two zones creates dramatic traffic congestion and travel time uncertainty, first on link 5 and then on link 4. For the base case, we can calculate the variance of path travel time estimates for these three OD pairs, respectively, as 1+2+2+3+4=12, 1+2=3 and 2+3+4=9, leading to a total path travel time estimation uncertainty as *TU* = 12×1+3×1+9×1 = 24.

In both cases (I) and (II), two AVI sensors are first installed at nodes *a* and *b*. In case (I), an additional AVI sensor is located at node *f* so that we can obtain two pairs of end-to-end travel time measurements: from node *a* to node *f*, and from node *f* to node *b*. The second measurement directly monitors travel time dynamics on link 5. In this particular example along the linear corridor, the end-to-end travel time statistics from *a* to *b* can be explicitly determined from the above two *mutually exclusive* observations. In order to avoid double-counting the information gain for the same data sources, the information quantification module in this study only considers two raw measurements: from *a* to *f*, and from *f* to *b*, to update the link travel time variance covariance matrix from  to. To do so, the measurement error matrix is assumed to be , and the mapping matrix , where the first measurement from *a* to *f* covers links 1,2,3 and 4, and the second measurement from *f* to *b* covers link 5. As link 5, with the highest travel time uncertainty, is directly measured from AVI readings, its link travel time estimate variance is reduced from 4 to 0.8, but the resulting  contains a large amount of correlation in its link travel time estimates for links 1 to 4. All the path travel time uncertainties for the three OD pairs have been reduced, and *TU* = 6.74.

In case (II), the third AVI sensor is installed at node *d* to match the underlying OD trip demand pattern, which produces sensor mapping matrix. The resulting *P*+ still contains two clusters of correlations corresponding to two individual measurements from *a* to *d* and from *d* to *b*. The path travel time estimate variances for the OD pairs from *a* to *d* and from *d* to *b* are dramatically reduced to 0.75 and 0.9, respectively. Although link 5 still has a relatively large estimate variance of 2.4 (its overall estimation error measure), the total travel time estimation uncertainty *TU* is now 3.3, which is much lower than *TU* = 6.74 in case (I).

In comparison, the analytical approach (31) does not consider the flow weight on each link, and results a decision value 0.52 for case (I) and 0.48 for case (II). For the targeted estimation uncertainty *P*+, case (I) has a log-determinant value of 0.065 and case (II) of 0.18, with the lower bound of the log-determinant value being -0.53.

This example shows that by locating and spacing AVI sensors to naturally match the spatial trip patterns of commuters, case (II) is able to systematically balance the trade-off between the needs for monitoring local traffic variations and end-to-end trip time dynamics. It is also important to notice that, both cases (I) and (II) have the same sensor network size and generate the same number of measurements every day, but with different amount of estimation uncertainty from a commuter/road user perspective. Thus, simple measures of information, such as traffic network coverage and the number of measurements, might not be able to quantify the system-wide uncertainty reduction and information gain for traveler information provision applications.



**Fig. 5**. Example of locating AVI sensors on a linear corridor

**6. Conceptual Framework and Data Flow**

Figure 6 illustrates the conceptual framework and data flow for sensor design problems. From sensor network design plans in block 1, we need to extract three groups of critical input parameters: AVI/AVL market penetration rates *α* and *β* at block 2, measurement error variance-covariance *R* in block 3, and sensor location mapping matrix *H* in block 4. Location mapping matrix *H* is derived from the sensor location sets , and .

The link travel time estimation module uses a Kalman filtering model to update mean travel time estimates (from block 6 to 7) and the corresponding estimation error variance matrix (from block 8 to 9), where the critical Kalman gain matrix *K*, calculated in block 5, is applied to the above two mean and variance propagation processes. Based on the estimation error variance statistics in block 9, the information quantification module derives the measure of information in block 10 by representing the path travel time estimation quality as a function of *P+*. By minimizing the network-wide path travel time estimation uncertainty, the sensor network design module finally selects and implements an optimized sensor plan so that point sensor, AVI, and AVL measurement data in block 12 can be produced from the actual sensor network, illustrated by block 11.

One of the key features offered by the Kalman Filtering model is that, although updating the travel time mean estimates from in block 6 to in block 7 requires sensor measurements *Y*, the uncertainty propagation calculation from block 8 and 9 (i.e. updating from ) does not rely on the actual sensor data, as the uncertainty reduction formula in block 9 is a function of three major inputs: a priori uncertainty matrix , measurement error range *R*, and sensor mapping matrix *H*. In other words, if a transportation analyst can reasonably prepare the above three input parameters, then he/she can apply the proposed analytical model to compute the information gain for a sensor design scenario and further assist the decision-maker to determine where and with what technologies sensor investments should be made in a traffic network.



Figure 6. Conceptual Framework and Data Flow For Sensor Network Design